

Learning Multi-level Sparse Representations for Identifying Neuronal Activity

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Abstract—Bilinear approximation of a matrix is a powerful paradigm of unsupervised learning. In some applications, however, there is a natural hierarchy of concepts that ought to be reflected in the unsupervised analysis, *e.g.* neurosciences image sequences. Therefore, we propose a decomposition of the matrix of observations into a product of more than two sparse matrices allowing for both hierarchical and heterarchical relations of lower-level to higher-level concepts. In addition, we *learn* the nature of these relations rather than imposing them. Finally, the proposed model yields plausible interpretations of the experimental neurosciences data (pixel \rightarrow neuron \rightarrow assembly), and fully recovers the structure from synthetic data that was modeled after the experiment.

I. INTRODUCTION

This work was stimulated by a concrete problem, namely the decomposition of state-of-the-art calcium imaging sequences into neurons, and assemblies of neurons [2]. Leveraging sparsity constraints seems natural, given that the neural activations are sparse in both space and time. The poor z-resolution of the data results each pixel can be assigned to more than one neuron. In addition, it is anticipated that one neuron can be part of more than one assembly.

A standard sparse decomposition of the set of vectorized images into a dictionary and a set of coefficients would not match prior knowledge that we have entities at three levels: the pixels, the neurons, and the assemblies, see Fig. 1. As a consequence, we propose a multi-level decomposition that 1) allows enforcing (structured) sparsity constraints at each level; 2) admits both hierarchical or heterarchical relations between levels (Fig. 1); 3) can be learned jointly, and 4) yields good results on real-world experimental data.

II. PROPOSED APPROACH

Given is a sequence of n noisy sparse calcium images which we vectorize and collect in the columns of matrix \mathbf{X} . We would like to find the following:

- a dictionary \mathbf{D} of q_0 vectorized images comprising m pixels each. Ideally, each basis function should correspond to a single neuron.
- a matrix \mathbf{A}^1 indicating to what extent each of the q_0 neurons is associated with any of the q_1 neuronal assemblies. This matrix encapsulates the quintessential structure we extract from the raw data, *viz.*, which lower-level concept is associated with which higher-level concept.
- a coefficient matrix $[\mathbf{U}^0]^T$ and $[\mathbf{U}^1]^T$ that encode in its rows the temporal evolution (activation) of the q_0 single neuron and the q_1 neuronal assemblies across n time steps, respectively.

Decomposing \mathbf{X} into constituent signals at different levels of representation, *e.g.* neurons and assemblies, is inferred jointly by minimizing the equation shown in Fig. 2. The Ω_U encourages sparsity of the coefficient matrix; whereas Ω_D prevents the inflation of dictionary entries to compensate for small coefficients, and induces, if desired, additional structure on the learned basis functions [1]. However, the problem is not *jointly* convex, but becomes convex w.r.t. one variable while keeping fixed the others if we assume that the norms Ω_U , Ω_D , and Ω_A are also convex. Hence, the main

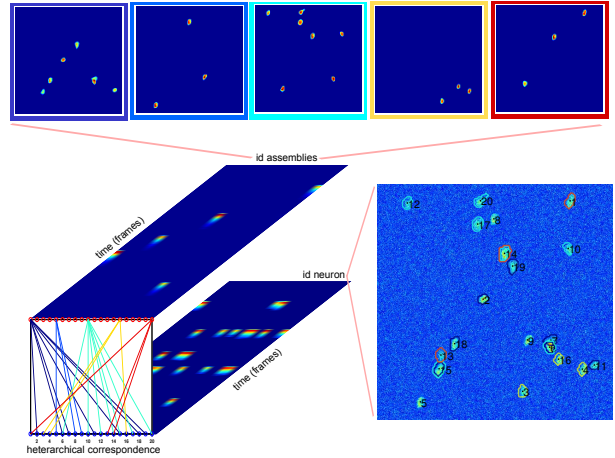


Fig. 1. Bottom left: Shown are the temporal activation patterns of individual neurons (lower layer), and assemblies of neurons (upper layer). Neurons and assemblies are related by a bipartite graph the estimation of which is a central goal of this work. The signature of five neuronal assemblies in the spatial domain is shown at the top. The bottom right shows the outline of all found neurons superimposed on a maximum intensity projection across the image sequence. All results shown in this figure issue from computations on a synthetic sequence, with known ground truth.

$$\min_{\mathbf{U}^0, \mathbf{U}^1, \mathbf{A}^1, \mathbf{D}} \frac{1}{2} \|\mathbf{X} - \mathbf{D}[\mathbf{U}^0]^T\|_F^2 + \frac{\eta_{D,1}^2}{2} \|\mathbf{X} - \mathbf{D}\mathbf{A}^1[\mathbf{U}^1]^T\|_F^2 + \frac{\eta_{U,1}^2}{2} \|\mathbf{U}^0 - \mathbf{U}^1[\mathbf{A}^1]^T\|_F^2 + \sum_{i=0}^n \sum_{t=1}^n \lambda_U^i \Omega_U(\mathbf{u}_t^i) + \sum_{j=1}^{q_1} \lambda_A^j \Omega_A(\mathbf{a}_j^i) \quad \text{s.t.} \quad \begin{cases} \forall k, \Omega_D(\mathbf{d}_{k,i}) \leq 1 \\ \forall i, \Omega_B(\mathbf{D}\mathbf{a}_i^1) \leq 1 \end{cases}$$

Fig. 2. Illustration of the main equation.

optimization procedure is based on iteratively optimizing a group of variables while fixing the others.

Our method achieves a sensitivity of 93% on average and discriminates robustly individual cells for different levels of correlated cell activity and of noise; whereas the accuracy of learning the latent structure \mathbf{A}^1 is 87%. In addition, our method is able to 1) identify and monitor neuronal activity at single cell and assembly level, 2) infer the assignment matrix behind the neuronal activity, and 3) distinguish highly correlated cells.

III. ACKNOWLEDGEMENT

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REFERENCES

- [1] R. Jenatton, G. Obozinski, and F. Bach. Structured sparse principal component analysis. In *International Conference on Artificial Intelligence and Statistics (AISTATS)*, 2010.
- [2] S. Reichinnek, A. von Kameke, A. M. Hagenston, and *et. al.* Reliable optical detection of coherent neuronal activity in fast oscillating networks in vitro. *NeuroImage*, 60(1), 2012.